ELSEVIER

Computers & Geosciences ▮ (▮▮▮▮) ▮▮▮–▮▮▮

COMPUTERS & GEOSCIENCES

www.elsevier.com/locate/cageo

# Trend-surface analysis of morphometric parameters: A case study in southeastern Brazil

Carlos Henrique Grohmann*

*Pós-Graduação, Instituto de Geociências, Universidade de São Paulo, Rua do Lago 562, 05508-800 Sao Paulo, SP, Brasil*

Received 14 March 2004; received in revised form 16 February 2005; accepted 24 February 2005

## Abstract

Trend-surface analysis was carried out on data from morphometric parameters isobase and hydraulic gradient. The study area, located in the eastern border of Quadrilátero Ferrífero, southeastern Brazil, presents four main geomorphological units, one characterized by fluvial dissection, two of mountainous relief, with a scarp of hundreds of meters of fall between them, and a flat plateau in the central portion of the fluvially dissected terrains. Morphometric maps were evaluated in GRASS-GIS and statistics were made on *R* statistical language, using the *spatial* package. Analysis of variance (ANOVA) was made to test the significance of each surface and the significance of increasing polynomial degree. The best results were achieved with sixth-order surface for isobase and second-order surface for hydraulic gradient. Shape and orientation of residual maps contours for selected trends were compared with structures inferred from several morphometric maps, and a good correlation is present.

*Keywords:* Trend-surface analysis; Morphometric analysis; Geographic information systems; GRASS-GIS; *R* statistical language; Analysis of variance

## 1. Introduction

Trend-surface analysis is the mathematical method used to separate map data into components of regional nature from local fluctuations (Agterberg, 1974; Davis, 1986). Regional trends are computed as polynomial surfaces of successive powers; residual values, corresponding to local fluctuations, are the arithmetic difference between original data and trend surface. Residuals maps play an important role in trend-surface analysis given that they can favour on identify or accentuate features of interest. This technique has been widely used by petroleum geologists, to predict structural behaviour of stratigraphic units in search for traps, geographical features, or to recognize structural "breaks" between successive units (Merriam and Harbaugh, 1963; Merriam and Lippert, 1966; Sutterlin and Hastings, 1986; Davis, 1986).

Morphometric maps are important tools in studies related to neotectonics and geomorphology, where the answers of natural landscapes to planet's interior dynamics are often masked by fast action of weathering, and the presence of drainage network anomalies and relief pattern discontinuities may be related with recent terrain movements (Zuchiewicz, 1991; Rodriguez, 1993; Salvador and Riccomini, 1995; Hiruma and Riccomini, 1999; Hiruma et al., 1999).

This work presents trend-surface analysis of the morphometric parameters, hydraulic gradient (Rodriguez, 1993) and isobase (Filosofov, 1960 cited in Jain,

---

*Tel.: +55 11 30914216; fax: +55 11 30914258.
*E-mail addresses:* guano@usp.br, carlos_grohmann@yahoo.com.br.

doi:10.1016/j.cageo.2005.02.011

1984; Golts and Rosenthal, 1993), with integrated utilization of free software GRASS-GIS (US Army CERL, 1993) and *R* statistical language (Ihaka and Gentleman, 1996), in the Serra do Caraça region, eastern border of Quadrilátero Ferrífero, southeastern Brazil.

## 2. Geological and geomorphological context

The Quadrilátero Ferrífero region, south São Francisco Craton, is characterized by Archaean granite-gneiss domed complexes coeval to Rio das Velhas greenstone belt (Machado et al., 1992), engaged to south and east in a Paleoproterozoic metamorphic belt (Mineiro belt, Teixeira and Figueiredo, 1991) defined as an acrescionary orogen of ca. 2.1 Ga (Fig. 1).

The study area presents four main geomorphological units (Fig. 2): *Serra do Caraça Range*, with average altitudes of 1400–1600 m and maximum at 2064 m; *Serra do Pinho Range*, in the eastern side of the area, with N–S trend; *Chapada de Canga Plateau*, in the central region of the study area, leveled at ca. 900 m; *Minas Gerais center-south and east Highlands*, characterized by fluvial dissection. The scarp that limits the Serra do Caraça from the other units has hundreds of meters of fall, and leads to believe that not only erosional processes, but also post-Cretaceous tectonic movements contributed in the morphological evolution.

According to Varajão (1991), remains of planation surfaces in the Quadrilátero Ferrífero have close relations with lithostructural domains. Reactivations with vertical displacement of ancient faults would be responsible for present-day altimetric differences (King, 1956; Barbosa, 1980).

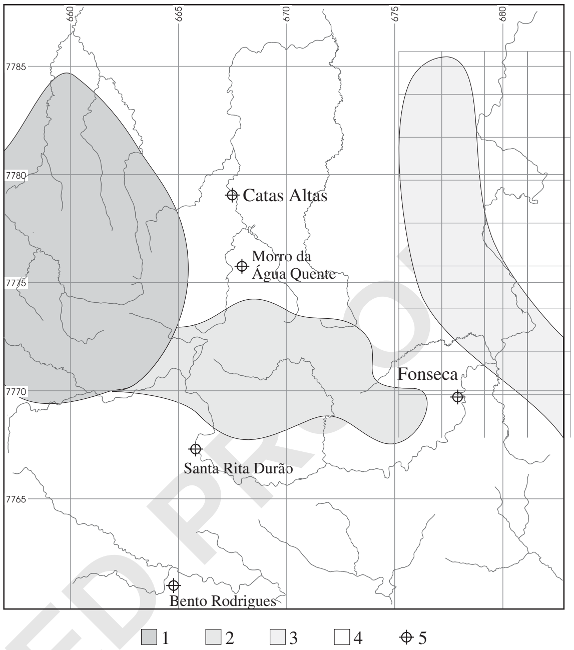

Fig. 2. Main geomorphological units in study area. (1) Serra do Caraça Range; (2) Chapada de Canga Plateau; (3) Serra do Pinho Range; (4) Minas Gerais center-south and east Highlands; and (5) Cities.

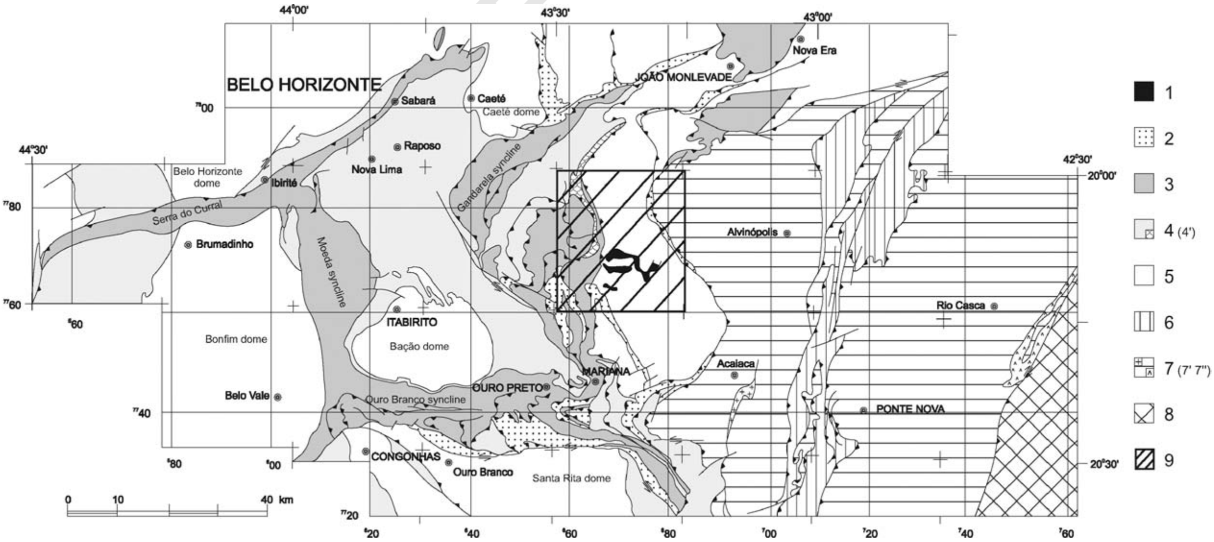

Fig. 1. Simplified geological map of southeast São Francisco craton (modified from Campos Neto et al., 2004). (1) Cenozoic Chapada de Canga and Fonseca Fms.; (2) Itacolomi/Espinhaço Gr.; (3) Minas Supergroup; (4) Rio das Velhas Supergroup (4′ – komatiitic unit); (5) SE São Francisco craton gneiss; (6) Dom Silvério Gr.; (7) Mantiqueira gneisses (7′ – amphibolite, 7″ – granulite); (8) Juiz de Fora granulites; and (9) Study area.

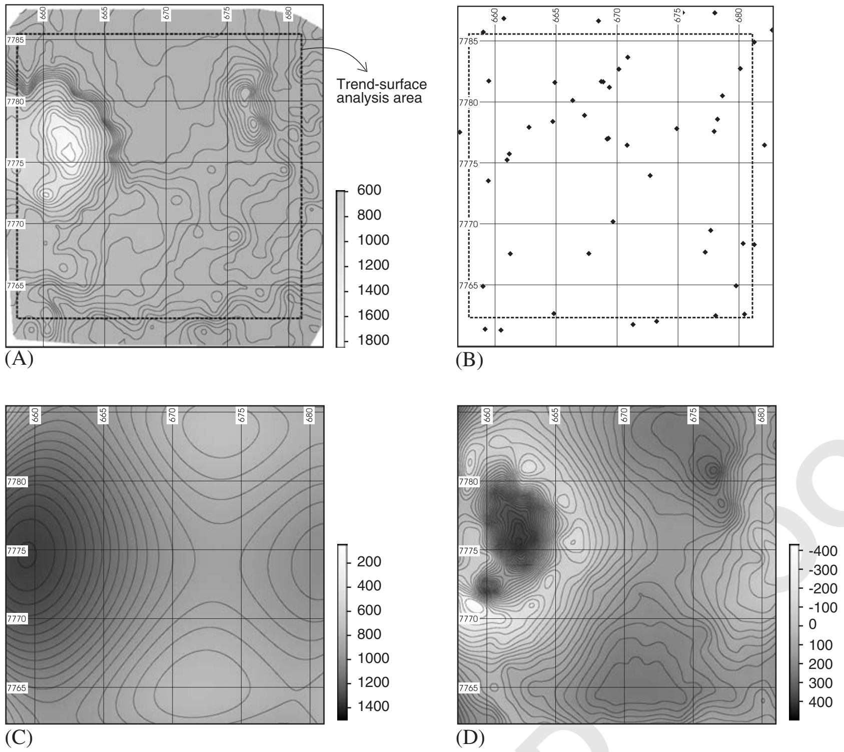

Fig. 3. (A) Original isobase surface, with area used for trend-surface analysis in dotted line; (B) data points, $n = 50$; (C) sixth-order trend surface; and (D) residuals of sixth order.

Evidences of Cenozoic tectonics in the Quadrilátero Ferrífero are observed in several sedimentary deposits and indicate three deformational events with distinct tension fields. The first event, extensive, oriented NNE–SSW and probably related with horsts and grabens oriented ESE–WNW. A second, and more expressive event, mainly compressive NW–SE. The third event is considered to be the relaxing of previous event structures (Lipski et al., 2001).

In the study area, the Quaternary Chapada de Canga Formation consists of a flat plateau formed by a succession of continental itabiritic ironstone pebble conglomerates and directly overlies the Eocene Fonseca Formation and the Precambrian basement (Sant'Anna and Schorscher, 1995). Cenozoic deposits are cut by NE and NW brittle faults and joints, related to reactivation of pre-existing structures in Precambrian basement, process that strongly influenced the development of present-day landscape morphology and drainage network (Sant'Anna et al., 1997).

## 3. Methods

The GIS used was the Geographic Resources Analysis Support System—GRASS 5.0.3 (US Army CERL, 1993; Neteler, 1998; Neteler and Mitasova, 2002; GRASS Development Team, 2002), an open-source project, freely available on the Internet,[1] which offers an integrated environment for raster and vector analysis, image processing and maps/graphics creation.

Statistical analysis were carried out on R, a system for statistical computation and graphics (Ihaka and Gentleman, 1996; Grunsky, 2002; R Development Core Team, 2003), through an interface with GRASS (Bivand, 2000) that allows raster maps and points files to be treated as variables for analysis. The *R* core package and extensions, as well as related documentation, can be obtained from CRAN (The Comprehensive *R* Archive Network).[2]

Morphometric parameters were developed in GRASS according to the propositions of Grohmann (2004).

Isobase lines (Filosofov, 1960 cited in Jain, 1984; Golts and Rosenthal, 1993) draw erosional surfaces; hence isobase surfaces are related to erosional cycles, mainly the most recent ones. The map of isobase (Fig. 3A) was made from interpolation of the intersections of

---

[1] Official GRASS-GIS homepage, http://grass.itc.it

[2] The Comprehensive R Archive Network, http://cran.r-project.org.

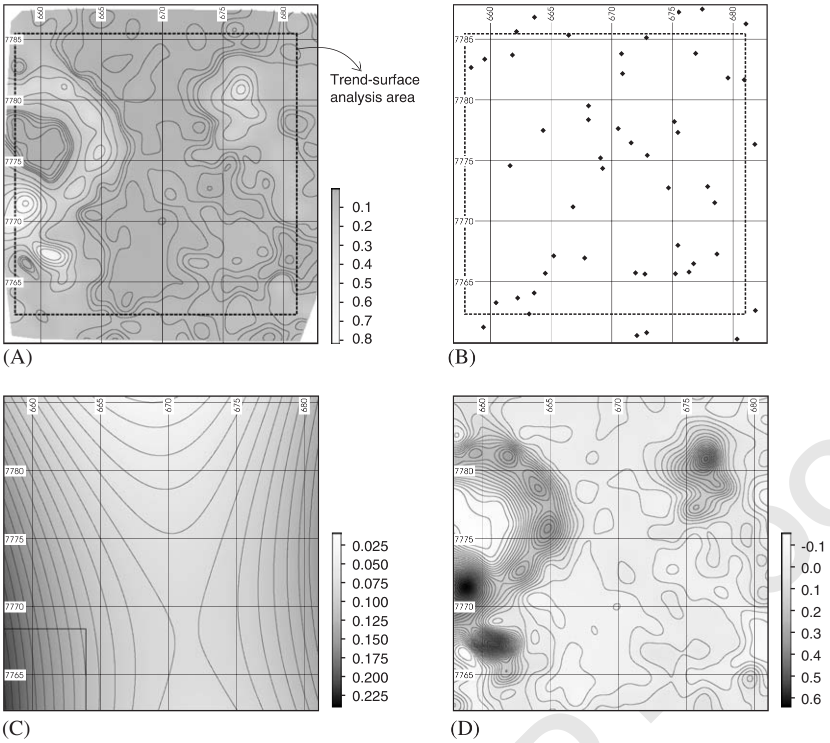

Fig. 4. (A) Original hydraulic gradient surface, with area used for trend-surface analysis in dotted line; (B) data points, $n = 50$; (C) second-order trend surface; and (D) residuals of second order.

contours with second- and third-order stream channels (drainage orders according to Strahler, 1952a, b).

The map of hydraulic gradient was elaborated to determine areas with similar hydraulic behaviour (Fig. 4A). This parameter is calculated for each second-order stream channel as the ratio of the altimetric difference between head and mouth with the plan length; the value is attributed to the mid-portion of the stream and point's values are interpolated.

Points files were imported into *R* as data frames; trend surfaces were computed using the *R* package *spatial*, which allows the fit of a polynomial surface up to sixth degree, and inserted back into GRASS as raster maps; residual surfaces were obtained by subtracting trends from the original surface.

A problem that can affect not only trend surfaces, but also contour maps, moving-average and other forms of fitted surfaces is the presence of "edge effects", which occur when there are few (or no) control points on the map boundary, so there are almost no constraints on the form of the surface (Davis, 1986; Landim, 1998). In order to avoid these effects, the area used for analysis is smaller than the area of the morphometric maps, assuring the presence of a "buffer", in which edge effects are concentrated (Figs. 3A and B, 4A and B).

The goodness-of-fit of trend surfaces can be statistically tested by analysis of variance (ANOVA), comparing the variance of regression to the variance of deviations (residuals). When a series of equations of successively higher degree are fitted to the data, this analysis can be expanded to analyse the contribution of additional partial regression coefficients and measure the appropriateness of increasing the order of equations (Davis, 1986). The general ANOVA table for testing the significance of increasing degree of polynomial surface is presented in Table 1.

The number of observations is very important in trend-surface analysis, due to its influence on the *F* test; if a large number is used, any test may present statistical significance. Since the original datasets for morphometric data analysed have 4512 points for isobase and 453 points for hydraulic gradient, 50 points were randomly selected from each dataset for analysis (Fig. 3B, 4B).

ANOVA data for evaluated morphometric parameters are presented in Table (2) for isobase and Table (3) for hydraulic gradient. All *F* values for isobase are very high, and the best correlation is of sixth-order surface (Fig. 3C), with an $R^2$ of 0.794. The best *F*-values

Table 1
General ANOVA for significance of increasing degree of a polynomial trend from $p$ to $(p+1)$ degree; polynomial equation of degree $p$ has $k$ coefficients, not counting $b_o$ term; equation of degree $(p+1)$ has $m$ coefficients, not counting $b_o$ term; number of observations is $n$

| Source of variation | Sums of squares | Degrees of freedom | Mean square | $F$ test |
|---|---|---|---|---|
| Regression of degree $p$ | $SS_{RP}$ | $k$ | $MS_{RP}$ | $MS_{RP}/MS_{DP}$ [a] |
| Deviation from degree $p$ | $SS_{DP}$ | $N-k-1$ | $MS_{DP}$ | |
| Regression of degree $(p+1)$ | $SS_{RP+1}$ | $m$ | $MS_{RP+1}$ | $MS_{RP+1}/MS_{DP+1}$ [b] |
| Deviation from degree $(p+1)$ | $SS_{DP+1}$ | $n-m-1$ | $MS_{DP+1}$ | |
| Regression due to increase from $p$ to $(p+1)$ degree | $SS_{RI}=SS_{RP+1}-SS_{RP}$ | $m$—$k$ | $MS_{RI}$ | $MS_{RI}/MS_{DP+1}$ [c] |
| Total variation | $SS_T$ | $n-1$ | | |

[a] Test of significance of $p$-degree trend surface.
[b] Test of significance of $(p+1)$-degree trend surface.
[c] Test of significance of increase in fit of $(p+1)$ degree over $p$ degree.

Table 2
General ANOVA for significance of increasing degree of isobase trend surfaces from first to sixth degree

| Degree | Source of variation | Sums of squares | Degrees of freedom | Mean square | $R^2$ | $F$ | $F_{(0.05)}$ |
|---|---|---|---|---|---|---|---|
| First | Regression | $8.15\times10^5$ | 2 | $4.1\times10^5$ | 0.242 | 8.823 | 3.195 |
| | Deviation | $21.7\times10^5$ | 47 | $0.5\times10^5$ | 0.758 | | |
| Second | Regression | $14.8\times10^5$ | 5 | $3\times10^5$ | 0.438 | 8.652 | 2.427 |
| | Deviation | $15\times10^5$ | 44 | $0.3\times10^5$ | 0.562 | | |
| *Regression due to Increase* | | $6.6\times10^5$ | 3 | $2.2\times10^5$ | | 6.477 | 2.816 |
| Third | Regression | $18.5\times10^5$ | 9 | $2.0\times10^5$ | 0.535 | 7.266 | 2.124 |
| | Deviation | $11.3\times10^5$ | 40 | $0.3\times10^5$ | 0.465 | | |
| *Regression due to Increase* | | $3.7\times10^5$ | 4 | $1.0\times10^5$ | | 3.285 | 2.606 |
| Fourth | Regression | $22.9\times10^5$ | 14 | $1.6\times10^5$ | 0.676 | 8.32 | 1.986 |
| | Deviation | $7.0\times10^5$ | 35 | $0.2\times10^5$ | 0.324 | | |
| *Regression due to Increase* | | $4.4\times10^5$ | 5 | $0.9\times10^5$ | | 4.498 | **2.485** |
| Fifth | Regression | $24.9\times10^5$ | 20 | $1.2\times10^5$ | 0.719 | 7.258 | 1.945 |
| | Deviation | $4.9\times10^5$ | 29 | $0.2\times10^5$ | 0.281 | | |
| *Regression due to Increase* | | $1.9\times10^5$ | 6 | $0.3\times10^5$ | | 1.874 | 2.432 |
| Sixth | Regression | $27.1\times10^5$ | 27 | $1.0\times10^5$ | 0.794 | 8.008 | 2.071 |
| | Deviation | $2.7\times10^5$ | 22 | $0.1\times10^5$ | 0.206 | | |
| *Regression due to Increase* | | $2.2\times10^5$ | 7 | $0.3\times10^5$ | | 2.523 | 2.464 |

for hydraulic gradient were achieved with second-order polynomial (Fig. 4C), with $R^2$ of 0.146.

## 4. Discussion

Morphometric analysis of the study area was carried out by Grohmann and Campos Neto (2003), where several parameters were employed to infer structures that can be related with landscape configuration; a morphological scenario was proposed, with a depressed central area enclosed by uplifted hills (Fig. 5). The presence of these structures can be seen on residual maps for the analysed parameters (Fig. 3D, 4D), in the change of shape and orientation of contours.

The second-order residuals for hydraulic gradient show a fast transition to higher values in the eastern region, with a strong alignment of contours in NW–SE and NE–SW directions, which mark the scarp of Serra do Caraça. The sixth-order residuals for isobase also have good agreement with inferred structures; in this map, a region of very negative deviations marks the scarp.

Table 3
General ANOVA for significance of increasing degree of hydraulic gradient trend surfaces from First to Sixth degree

| Degree | Source of variation | Sums of squares | Degrees of freedom | Mean square | $R^2$ | *F* | F(0.05) |
|---|---|---|---|---|---|---|---|
| First | Regression | 0.014 | 2 | 0.007 | 0.016 | 1.400 | 3.195 |
| | Deviation | 0.241 | 47 | 0.005 | 0.984 | | |
| Second | Regression | 0.060 | 5 | 0.012 | 0.146 | 2.680 | 2.427 |
| | Deviation | 0.195 | 44 | 0.004 | 0.854 | | |
| *Regression due to Increase* | | 0.045 | 3 | 0.015 | | 3.391 | 2.816 |
| Third | Regression | 0.070 | 9 | 0.008 | 0.108 | 1.659 | 2.124 |
| | Deviation | 0.190 | 40 | 0.005 | 0.892 | | |
| *Regression due to Increase* | | 0.010 | 4 | 0.002 | | 0.5267 | 2.606 |
| Fourth | Regression | 0.097 | 14 | 0.007 | 0.133 | 1.540 | 1.986 |
| | Deviation | 0.158 | 35 | 0.004 | 0.867 | | |
| *Regression due to Increase* | | 0.028 | 5 | 0.005 | | 1.235 | 2.485 |
| Fifth | Regression | 0.114 | 20 | 0.006 | 0.063 | 1.167 | 1.945 |
| | Deviation | 0.142 | 29 | 0.005 | 0.937 | | |
| *Regression due to Increase* | | 0.016 | 6 | 0.002 | | 0.563 | 2.432 |
| Sixth | Regression | 0.157 | 27 | 0.006 | 0.140 | 1.294 | 2.071 |
| | Deviation | 0.099 | 22 | 0.004 | 0.860 | | |
| *Regression due to Increase* | | 0.043 | 7 | 0.006 | | 1.367 | 2.464 |

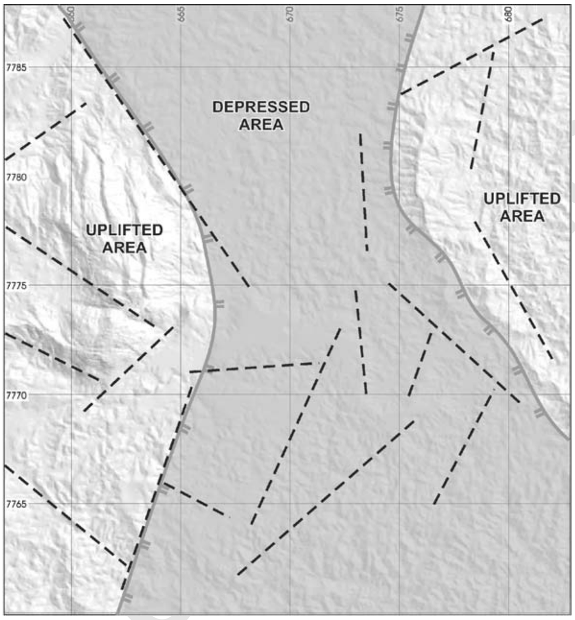

Fig. 5. Morphological scenario and inferred structures for study area, according to Grohmann and Campos Neto (2003).

There is a nearly E–W structure in the central portion of the study area, admitted as a normal fault (Sant'Anna et al., 1997) associated with a morphological step of the Chapada de Canga plateau, which is well marked in swath profiles (Fig. 6) made by Grohmann (2004).

Despite the fact that the Chapada de Canga plateau is a very distinctive topographic feature, it does not show much expression on morphometric maps, which can be explained by the fact that it is an area of very low drainage density, and that evaluated morphometric parameters are based on the relations of topography with drainage network.

## 5. Conclusions

Trend-surface analysis was applied to morphometric data, and ANOVA was used to determine the most representative polynomial surface. The residuals maps were compared with a proposed morphological scenario, and there is a good correlation between inferred structures with shape and orientation of contours. In the central portion of the area, there is a flat plateau, which has little expression in morphometric maps, although it represents a distinct topographic feature. This can be seen as a result of low drainage density in the flat area, since most morphometric methods are based on relations of topography with drainage network.

The fit of trend surfaces with very different orders (sixth and second) for morphometric parameters of the same area, may be explained by the spatial behaviour of these parameters. The isobase method tends to produce a smooth surface across the area, while hydraulic gradient is more sensitive to local fluctuations, and

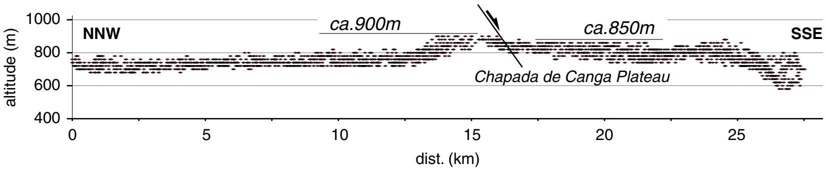

Fig. 6. Swath profile oriented NNW–SSE through the center of study area (Grohmann, 2004). Note step of flat plateau in its mid-portion, associated with a normal fault.

creates a surface with bigger variability, which could be better studied with frequency analysis methods, such as Fourier series.

## Acknowledgements

The author expresses sincere gratefulness to those that, directly or indirectly, had collaborated for the accomplishment of this work: Mario da Costa Campos Neto, for orientation and incentive; Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), for M.Sc. scholarship cession (proc. 130777/2002-1); Claudio Riccomini and Carlos Roberto de Souza Filho for discussions; Jorge Kazuo Yamamoto, for help with spatial statistics; John Doveton and an anonymous reviewer for revising the manuscript.

## References

Agterberg, F.P., 1974. Geomathematics. Elsevier, Amsterdam 596p.

Barbosa, G.V., 1980. Superfícies de erosão no Quadrilátero Ferrífero. Revista Brasileira de Geociências 10, 89–101.

Bivand, R.S., 2000. Using the R statistical data analysis language on GRASS 5.0 GIS database files. Computers & Geosciences 26 (9/10), 1043–1052.

Campos Neto, M.C., Basei, M.A.S., Vlach, S.R.F., Caby, R., Szabó, G.A.J., Vasconcelos, P., 2004. Migração de orógenos e superposição de orogêneses: um esboço da Colagem Brasiliana no sul do Cráton do São Francisco, SE-Brasil. Geologia USP (Universidade de Sao Pãolo) 4 pp. 13–40.

Davis, J.C., 1986. Statistics and Data Analysis in Geology, second ed. Wiley, New York, NY 656pp.

Filosofov, V.P., 1960. Brief Guide to Morphometric Methods in Search of Tectonic Structures. Saratov University Publishing House (in Russian).

Golts, S., Rosenthal, E., 1993. A morphotectonic map of the northern Arava in Israel, derived from isobase lines. Geomorphology 7, 305–315.

GRASS Development Team, 2002. GRASS 5.0 Users Manual. ITC-irst, Trento, Italy. Electronic document: http://grass.itc.it/gdp/html_grass5/

Grohmann, C.H., 2004. Morphometric analysis in Geographic Information Systems: applications of free software GRASS and R. Computers & Geosciences 30 (9/10), 1055–1067.

Grohmann, C.H., Campos Neto, M.C., 2003. Serra do Caraça (Quadrilátero Ferrífero, MG): morphometric parameters and preliminary tectonic considerations. In: IX Simpósio Nacional de Estudos Tectônicos/III International Symposium on Tectonics, Armação de Búzios, pp. 261–262.

Grunsky, E.C., 2002. R: a data analysis and statistical programming environment – an emerging tool for the geosciences. Computers & Geosciences 28 (10), 1219–1222.

Hiruma, S.T., Riccomini, C., 1999. Análise morfométrica em neotectônica: o exemplo do Planalto de Campos do Jordão, SP. Revista do Instituto Geológico 20, 5–19.

Hiruma, S.T., Riccomini, C., Modenesi-Gauttieri, M.C., 1999. Neotectônica no Planalto de Campos de Jordão, SP. Revista Brasileira de Geociências 31, 375–384.

Ihaka, R., Gentleman, R., 1996. R: a language for data analysis and graphics. Journal of Computational and Graphical Statistics 5, 299–314.

Jain, V.E., 1984. , vol. 42. Geotectónica General. Mir Editions, Moscow, Russia 357pp.

King, L., 1956. Geomorfologia do Brasil Oriental. Revista Brasileira de Geografia 18, 147–266.

Landim, P.M.B., 1998. Análise estatística de dados geológicos. Fundação Editora da UNESP, São Paulo 226pp.

Lipski, M., Endo, I., Castro, P.T.A., Trzazkos-Lipski, B., 2001. Estudo do campo de tensões Cenozóicas no Quadrilátero Ferrífero, Minas Gerais. In: VIII Simpósio Nacional de Estudos Tectônicos, Recife, pp. 331–333.

Machado, N., Noce, C.M., Ladeira, E.A., Oliveira, O.A.B., 1992. U-Pb Geochronology of Archaean magmatism and Proterozoic metamorphism in the Quadrilátero Ferrífero, southern São Francisco craton, Brazil. Geological Society of America Bulletin 104, 1221–1227.

Merriam, D.F., Harbaugh, J.W., 1963. Computer helps map oil structures. Oil and Gas Journal 61, 158–163.

Merriam, D.F., Lippert, R.H., 1966. Geologic model studies using trend-surfaces analysis. Journal of Geology 74, 344–357.

Neteler, M., 1998. Introduction to GRASS GIS software. Hannover, Germany, available at http://www.geog.uni-hannover.de/grass/gdp/neteler/manual98.html

Neteler, M., Mitasova, H., 2002. Open Source GIS: A GRASS GIS Approach. Kluwer Academic Publishers, Boston, MA 464pp.

R Development Core Team, 2003. R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria, ISBN: 3-900051-00-3, http://www.R-project.org.

Rodriguez, S.K., 1993. Neotectônica e sedimentação quaternária na região de "Volta Grande" do Rio Xingu, Altamira, Pará. M.Sc. Thesis, Instituto de Geociências, Universidade de São Paulo, São Paulo, Brasil, 160pp.

Salvador, E.D., Riccomini, C., 1995. Neotectônica de região do alto estrutural de Queluz (SP-RJ, Brasil). Revista Brasileira de Geociências 25, 151–164.

Sant'Anna, L.G., Schorscher, H.D., 1995. Leques aluviais cenozóicos do flanco leste da Serra do Caraça, MG: a Formação Chapada de Canga. Anais da Academia Brasileira de Ciências 67, 519.

Sant'Anna, L.G., Schorscher, H.D., Riccomini, C., 1997. Cenozoic tectonics of the Fonseca Basin region, eastern Quadrilátero Ferrífero, MG, Brazil. Journal of South American Earth Sciences 10, 275–284.

Strahler, A.N., 1952a. Dynamic basis of geomorphology. Bulletin of the Geological Society of America 63, 923–938.

Strahler, A.N., 1952b. Hypsometric (area-altitude) analysis of erosional topography. Bulletin of the Geological Society of America 63, 1117–1142.

Sutterlin, P.G., Hastings, J.P., 1986. Trend-surface analysis revisited – a case history. Computers & Geosciences 12 (4B), 537–562.

Teixeira, W., Figueiredo, M.C.H., 1991. An outline of Early Proterozoic crustal evolution in the São Francisco craton, Brazil: a review. Precambrian Research 53, 1–22.

US Army CERL, 1993. GRASS 4.1 Reference Manual. US Army Corps of Engineers, Construction Engineering Research Laboratories, Champaign, IL, pp. 1–425.

Varajão, C.A.C., 1991. A questão da correlação das superfícies de erosão do Quadrilátero Ferrífero, Minas Gerais. Revista Brasileira de Geociências 21, 138–145.

Zuchiewicz, W., 1991. On different approaches to neotectonics: a Polish Carpathians example. Episodes 14, 116–124.